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A Depletion Protocol for Non-Renewable Natural Resources: Australia as an Example

Albert A. Bartlett[1,2]

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This paper examines the implications of statements by Australia's Minister of... Resources that Australia's exports of coal are growing rapidly and that Australia's coal will last "110 years at current rates of production." If one assumes that coal production $P(t)$, follows a Gaussian curve (similar to a Hubbert curve) one can construct a family of Gaussian curves showing possible future paths of $P(t)$ which are consistent with the cited "110 years." Each curve reaches a maximum after which $P(t)$ declines toward zero. Knowledge of the present value of dP/dt allows one member of the family to be identified as the most probable future path of $P(t)$. Families of curves and tabular data are presented for resource quantities that would last 50, 100 and 200 years "at current rates of production." If, instead, Australia's $P(t)$ follows a declining exponential curve $(\exp(-kt))$ with $k = (1/110)$ per year, the stated quantity of coal will allow production to continue forever, with $P(t)$ declining with a half life of 76 y. This and more rapidly declining exponential paths are the only paths that can be said to be sustainable. The envelope of the family of Gaussian curves divides the (P, t) plane into "allowed" and "forbidden" areas. The declining exponential curve divides the "allowed" area into an upper area that is "terminal" and a lower area that is "sustainable." These facts, coupled with Australia's expectations of rapid growth of its population, suggest that Australia's present resource policies are "anti-sustainable" and that the people of Australia need to rethink their present policy of rapidly exporting their fossil fuels.

KEY WORDS: Australia, energy policy, fossil fuels, greenhouse gases, Hubbert curve, population.

INTRODUCTION

The Australian Minister of Industry, Tourism, and Resources recently wrote:

> "Australia is the world's largest exporter of coal... Our demonstrated recoverable economic resource for black coal is estimated to be more than 39 billion tones, *or more than 110 years' worth of supply at current rales of production*... The Australian export coal industry... has *expanded production at the rate of around 5 per cent per annum over the last 20 years*... In 2003, Australia was the fourth-largest liquefied natural gas (LNG) exporter in the Asia-Pacific region and the seventh-largest in the world, exporting 8 million tonnes of LNG. Australia's LNG export capacity could potentially rise further to more than 50 million tonnes per annum (Mtpa) by early next decade... *The reserve-to-production ratio [for LNG] in Australia delivers an expected resource lifetime of approximately 77 years.*" (All emphasis has been added) (Macfarlane, 2005)

This quotation is representative of the optimistic view of the future of Australia and its energy resources as expressed by the Minister. The editor of the journal in which the Minister's article appeared wrote: The Minister's "focus is the *growth* and *sustainability* of Australian industries..." (emphasis added).

The goal of this paper is to make a mathematical examination of the hypothetical possible future paths of the rate of production vs. time of

[1]Department of Physics, University of Colorado, Boulder, CO 80309-0390, USA.
[2]To whom correspondence should be addressed; e-mail: albert.bartlett@colorado.edu.

a nonrenewable resource to see if the Minister's optimism is warranted by the facts.

DEFINITION OF TERMS

We define needed quantities:

t = time in years; the present time is designated as $t = 0$.
$R(i)$ = the quantity of the resource initially available before any of it was extracted.
$R(0)$ = the quantity of the resource remaining at time $t = 0$ in units such as tonnes. The value of $R(0)$ for specific resources is debatable and cannot be known exactly.
$R(t)$ = an estimate of the quantity of the resource remaining as a function of time, in units such as tonnes.
$P(t)$ = the annual rate of production of the resource as a function of time in units such as tonnes/y
$P(0)$ = the annual rate of production at the time $t = 0$ in tonnes/y.
k = the fractional annual growth rate of $P(t)$ in units of t^{-1}; if k is positive and constant, one has exponential growth. If k is negative and constant one has exponential decay.
$t(E)$ = is the abrupt expiration time of the reserves $R(0)$, "at current rates of production."
σ = the standard deviation of the Gaussian Error Curve.
R/P = is the reserves to production ratio (R/P ratio). It is the hypothetical estimate of the life-expectancy (years) of the reserves of a resource "at current rates of production." The ratio will change as resources are consumed or as new reserves become available.
$R(0)/P(0)$ = the value of the R/P ratio at time $t = 0$
$R(0)/P(0) = t(E)$

THE RESERVE TO PRODUCTION RATIO; *R/P*

When people wish to convey an idea of the life expectancy of reserves of nonrenewable resources they may give the life expectancy of the reserves "at current rates of production" which is determined by dividing the current estimate of the reserves $R(0)$ (in tonnes) by the current rate of production $P(0)$ (in tonnes/y). This is termed the "Reserve-to-Production Ratio," R/P which has the units of time (years). The life expectancy given by the R/P ratio is what one would calculate if $P(t)$ remains unchanged from its present value $P(0)$ until $R(0)$ is completely consumed (Bartlett, 1978). If $P(t)$ is growing, the life-expectancy of a resource will be less than it would be if $P(t)$ continued unchanged "at current rates of production." Yet even when $P(t)$ is growing, this "zero growth" parameter, R/P, is frequently cited and accepted as the life-expectancy of a resource. When $P(t)$ is growing, this use of the R/P ratio conveys the optimistic expectation of a life-expectancy that is longer than is warranted by the facts. This serious misrepresentation of the facts is widespread. It gives rise to unwarranted optimism on the part of political leaders, journalists and others.

For given values of $R(0)/P(0)$ of a resource, we want to examine, in generic terms, the limited scenarios of the $P(t)$ vs. t that are allowed by the numbers, and the even more limited scenarios that are sustainable.

HUBBERT CURVES

M. King Hubbert observed that the long-term history of the production $P(t)$ (tonnes/y) of a resource vs. time starts at zero, rises to one or more maxima, and then returns to zero. Hubbert (1974) The area under the complete curve represents $R(i)$ the total tonnes of the resource before any of it was extracted. Hubbert used the derivative of the Logistic function as a model for the rise and fall of $P(t)$. In a curve-fitting analysis of U.S. and world oil production data, Bartlett (2000) reported:

> "The analysis... was done with both curves (the derivative of the Logistic curve and the Gaussian curve) to allow comparison of the results. The differences in the results were smaller than the root mean square deviations of the fits, so that the results did not indicate a clear preference for either curve. Both curves are widely understood, but the Gaussian curve was used because the analysis seemed simpler in execution and interpretation."

Technically then, only the derivative of the Logistic function should be termed a Hubbert curve. Because the two curves are indistinguishable at the level of precision of the analysis of typical production data, the Gaussian curve is a good approximation to a Hubbert curve and is used here.

The results of calculations will be presented numerically and graphically for three representative values, 50, 100, and 200 years, of the R/P

life expectancy of resources "at current rates of production." Interpolation or extrapolation then can be done to estimate the hypothetical future scenarios of $P(t)$ for resources for which the current value of R/P has been estimated, including the two examples from Australia.

CALCULATIONS

We assume that the Gaussian Error Curve is a reasonable approximate scenario for the curve of $P(t)$ vs. t for finite nonrenewable resources. We will refer to the Gaussian scenarios as the "business as usual" scenarios. For each of the three assumed values of the $R(0)/P(0)$ ratio (50, 100, and 200 years) a generic family of Gaussian curves of differing widths is created which all start with $P(0) = 100$ tonnes per year at the present time, $t = 0$. Each curve rises to a maximum and then declines toward $P(t) = 0$. The curves differ from one another by having different standard deviations, σ, (widths). The area under the curve of $P(t)$ vs. t from $t = 0$ to t approaching ∞ is the quantity of the resource remaining at $t = 0$, so all members of a family of curves must have the same area between $t = 0$ and $t = \infty$. Each member of a family of curves has its own fractional rate of growth; $k = ((1/P)dP/dt)$, at $t = 0$. From the family, one picks the curve whose initial value of k equals the reported current fractional rate of increase of $P(t)$. This curve then can be considered to be the best estimate of the probable future path of $P(t)$ for the given $R(0)/P(0)$ ratio and for the "business as usual" scenario (Gaussian Curve).

The scenarios are modeled on an Excel spreadsheet which was used for all of the calculations and plotting.

UNCERTAINTIES IN THE CALCULATIONS

The largest uncertainty in these results arises from the uncertainty of our knowledge of the value of $R(0)$. Smaller uncertainties arise from the goodness of the fit of a Gaussian Curve to data for P from the real world. As an example, using data for the $P(t)$ vs. t of U.S. petroleum production, the root mean square (RMS) deviation between the data and the best-fit Gaussian was 3.2% of the height of the Gaussian maximum. (Bartlett, 2000)

EXPONENTIAL GROWTH OF $P(t)$

"At the current rate of production" implies the unrealistic scenario of $k = 0$, in which $P(t) = P(0)$ until the last tonne of the resource is produced, whereupon $P(t)$ falls abruptly to zero at the time $t(E)$ which is equal to the ratio $R(0)/P(0)$. The graph of the $P(t)$ vs. time is a rectangle whose height is $P(0)$; whose length is $R(0)/P(0)$ and whose area is $R(0)$.

However, if the rate of production $P(t)$ is growing at some fractional rate $k > 0$, such as 5% per year, $P(t) = P(0)\exp(kt)$, then it is enlightening to see what happens if one assumes that the fractional growth continues at an unchanged rate until all of the reserves $R(0)$ have been consumed at an expiration time $t(k)$, whereupon $P(t)$ falls abruptly to zero. This also is an unrealistic scenario, but it must be examined because our national and global economies are predicated on unending steady growth of our economies and of our rates of consumption of resources. The expiration time $t(k)$ is given by Robiscoe (1973),

$$t(k) = (1/k)\ln((kR(0)/P(0)) + 1) \qquad (1)$$

At the peak, just before expiration, the rate of production $P(t)$ is larger than the initial rate $P(0)$ by a factor:

$$P(t(k))/P(0) = ((kR(0)/P(0)) + 1) \qquad (2)$$

Representative expiration times for five values of $R(0)/P(0)$ and for different constant rates of growth of $P(t)$ are given in Table 1. For $P(0) = 100$ tonnes/y, the corresponding peak heights just before expiration are given in Table 2. For example, if a resource would last 200 years at present rates of production, Table 1 shows that with a constant growth of 5% per year, the resource will expire in 48 years. Table 2 shows that just before the resource expires $P(t)$ is 11 times $P(0)$. Several such scenarios $P(t)$ vs. t for $R(0)/P(0) = 200$ y are shown in Figure 1.

On the subject of LNG, the Minister writes optimistically that the $R(0)/P(0)$ ratio for Australia's (LNG) gives an "expected resource lifetime of approximately 77 years," and he reports that LNG exports are expected to grow from 8 million tonnes/y to 50 million tonnes/y by early in the next decade. If one assumes the expected growth takes place over a period of 10 years, then the average rate of growth of exports of LNG projected by the Minister is: (Bartlett, 1993)

$$k = (1/10)\ \ln(50/8) = 0.18 \text{ or } 18\% \text{ per year} \qquad (3)$$

Table 1. Expiration Times of a Nonrenewable Resource, Expressed in Years, for a Series of Reserve to Production Ratios, $R(0)/P(0)$, for Each of Several Constant Annual Growth Rates from 1 to 10% per Year, in Accord with the Patterns of Figure 1 and Equation (1)

$R(0)/P(0)$	Annual Percent Growth Rates				
(Years)	1	3	5	7	10
10	9.5	8.7	8.1	7.6	6.9
20	18.2	15.7	13.9	12.5	11.0
30	26.2	21.4	18.3	16.2	13.9
50	40.5	30.5	25.1	21.5	17.9
70	53.1	37.7	30.1	25.4	20.8
100	69.3	46.2	35.8	29.7	24.0
200	109.9	64.9	48.0	38.7	30.4
300	138.6	76.8	55.5	44.2	34.3
500	179.2	92.4	65.2	51.2	39.3

If one imagined that this growth rate could be continued until the cited amount of Australia's natural gas was completely extracted, Equations (1) and (2) show that the resource would expire in 15 years rather than 77 years cited by the Minister, and the peak production $P(t)$ would be 15 times $P(0)$.

CLOSER TO THE REAL WORLD

Even though it is consistent with accepted economic goals, it is unrealistic to imagine that an economy could maintain a constant unchanging $P(t)$ until the last bit of a resource has been extracted. To be more realistic we note that positive rates of growth of $P(t)$ will cause the graph of $P(t)$ vs. t to pass through the maximum after which $P(t)$ will decline and approach zero. We can model this behavior mathematically by assuming that $P(t)$ follows a Gaussian and that $dP/dt > 0$ at $t = 0$ so that the maximum is in the future. The curve of $P(t)$ vs. t is assumed to be given by:

$$P = P(0)\exp(-0.5(t/\sigma)^2) \quad (4)$$

where $P(0)$ will be assumed to be 100 tonnes/y, the time t is expressed in years, and σ is the standard deviation of the Gaussian curve in years.

Table 2. Peak Heights as Multiples of $P(0)$ for a Series Reserve to Production Ratios, $R(0)/P(0)$, for Each of Several Steady Annual Growth Rates from 1 to 10% per Year from Equation (2) and Figure 1

$R(0)/P(0)$	Annual Percent Growth Rates				
(Years)	1	3	5	7	10
10	1.1	1.3	1.5	1.7	2.0
20	1.2	1.6	2.0	2.4	3.0
30	1.3	1.9	2.5	3.1	4.0
50	1.5	2.5	3.5	4.5	6.0
70	1.7	3.1	4.5	5.9	8.0
100	2.0	4.0	6.0	8.0	11.0
200	3.0	7.0	11.0	15.0	21.0
300	4.0	10.0	16.0	22.0	31.0
500	6.0	16.0	26.0	36.0	51.0

RESULTS THAT CAN BE APPLIED TO THE REAL WORLD

In order to provide a basis for applying these results to a variety of resource situations, families of Gaussian curves for values of $R(0)/P(0) = 50$ y, 100 y, and 200 y are shown respectively in Figures 2–4. In addition to the family of curves, each figure has a rectangular curve that ends at the year $R(0)/P(0)$ which represents P continuing "at current rates of production," i.e. $k = 0$. The individual members of each family of curves are characterized by different standard deviations σ, and each curve in a given family has its own initial rate of growth of $P(t)$ at $t = 0$. All of the curves have the same area, $R(0)$ which is equal to $100(R(0)/P(0))$ tonnes. Data from the calculations for each of these situations are summarized in Tables 3–8.

EXAMPLE

For the situation of $R(0)/P(0) = 200$ y, Figure 4 shows the family of curves of $P(t)$ vs. t. The parameters of these curves are summarized in Tables 7 and 8. For $\sigma = 30$ y, Table 7 shows that at $t = 0$, $P(t)$ is growing 4.8 percent per year, the peak will be reached in year 44 and its height will be 2.86 times $P(0)$. In year 87, $P(t)$ will have fallen to its initial value of 100 tonnes/y and by symmetry, it will be falling at 4.8 percent per year. Table 8 shows that for $\sigma = 30$ years, it will take 47 years to consume 50% of $R(0)$. At the peak of the curve, the R/P ratio for the then remaining resource will be 38 years and in year 87 when P has dropped to its initial value of 100 tonnes/y, the R/P ratio will be 16 years. This example describes a rapid extraction and depletion of the resource.

Now let's find a curve that approximately fits the data given for Australia's LNG: $R(0)/P(0) = 77$

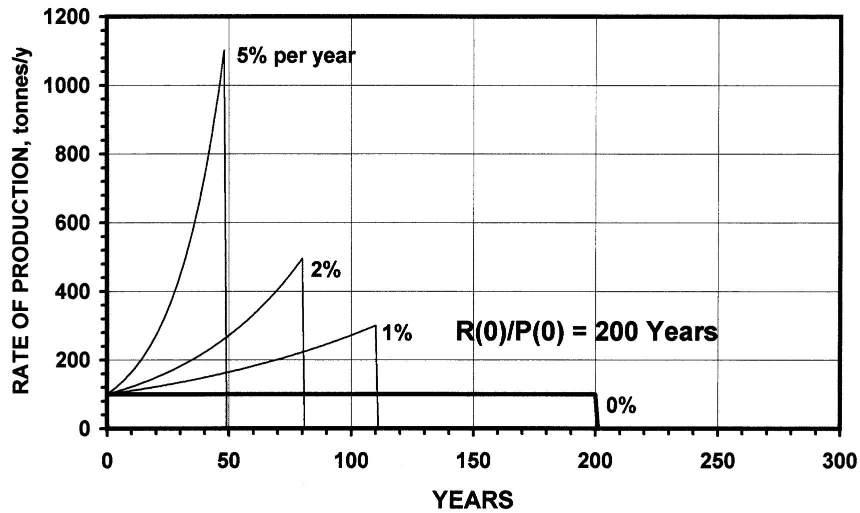

Figure 1. Generic graph of rate of production $P(t)$ vs. t. For all curves initial production at time $t = 0$ is 100 tonnes/y. If Reserves to Production ratio $R(0)/P(0) = 200$ years, then implied graph of $P(t)$ vs. time is given by lower horizontal line graph labeled "0%." If rate of growth of $P(t)$ is constant 5% per year until resource is consumed completely, then $P(t)$ vs. time is shown by sharply rising upper curve where peak rate of production is 11 times initial $P(0)$, and $P(t)$ then falls abruptly to zero in just under 50 years. Similar scenarios are shown for steady growth of $P(t)$ of 2% and 1% per year. Four curves all have same area of 20,000 tonnes. Data from these curves are included in Tables 1 and 2. These scenarios are illustrative but completely unrealistic.

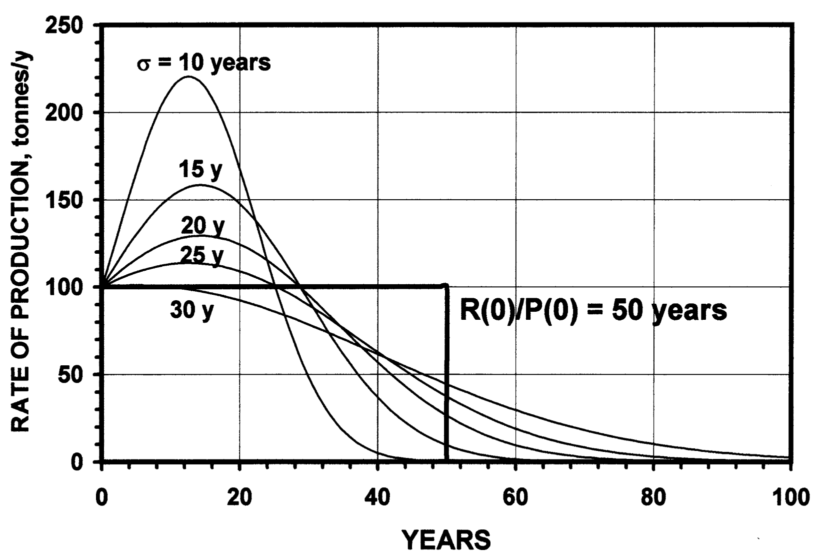

Figure 2. Generic graph of curves of $P(t)$ in vs. t for $R(0)/P(0) = 50$ years. Horizontal straightline shows a constant $P(t)$ of 100 tonnes/y for 50 years at which point the resource is consumed completely and $P(t)$ falls abruptly to zero. Other curves are "business as usual" Gaussian scenarios with values of the standard deviation σ of curves of (from top to bottom) 10 y, 15 y, 20 y, 25 y, and 30 y. Curves all have same area of $R(0) = 5000$ tonnes. Numerical data from this family of curves are given in Tables 3 and 4.

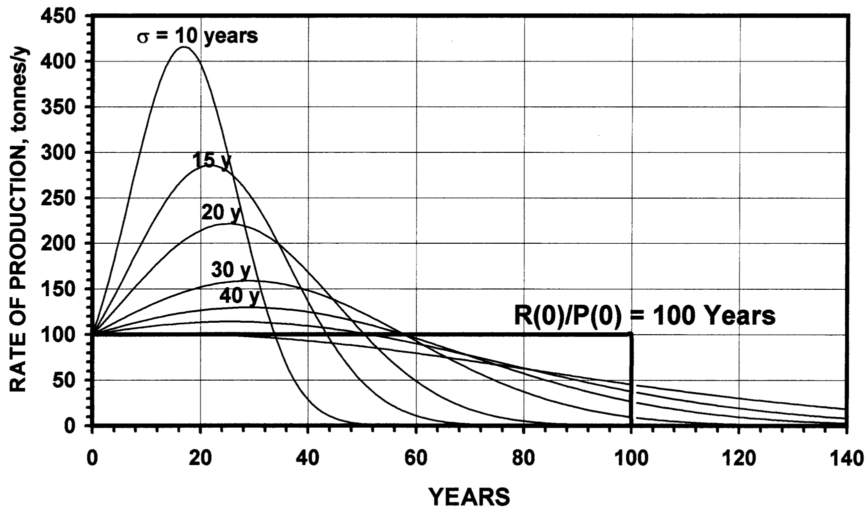

Figure 3. Generic graph of curves for $P(t)$ vs. t for case of $R(0)/P(0) = 100$ years. Horizontal curve shows constant $P(t)$ of 100 tonnes/y for 100 years at which point resource is consumed completely and $P(t)$ falls abruptly to zero. Other curves are "business as usual" Gaussian scenarios for values of standard deviation σ of curves of (from top to bottom) 10 y, 15 y, 20 y, 30 y, 40 y, 50 y, and 60 y. Curves all have same area of $R(0) = 10{,}000$ tonnes. Numerical data from this family of curves are given in Tables 5 and 6.

years and $((1/P)dP/dt) = 18\%$ per year. An approximate estimate of the future production scenario can be determined by using Figure 3 and Tables 5 and 6 which are for the $R(0)/P(0)$ of 100 years. In Table 5 we will use the numbers for $\sigma = 10$ y (initial slope = 20 percent per year) in place of the 18 percent per year from the Minister's statement. The tables indicate the following approximate results: the peak

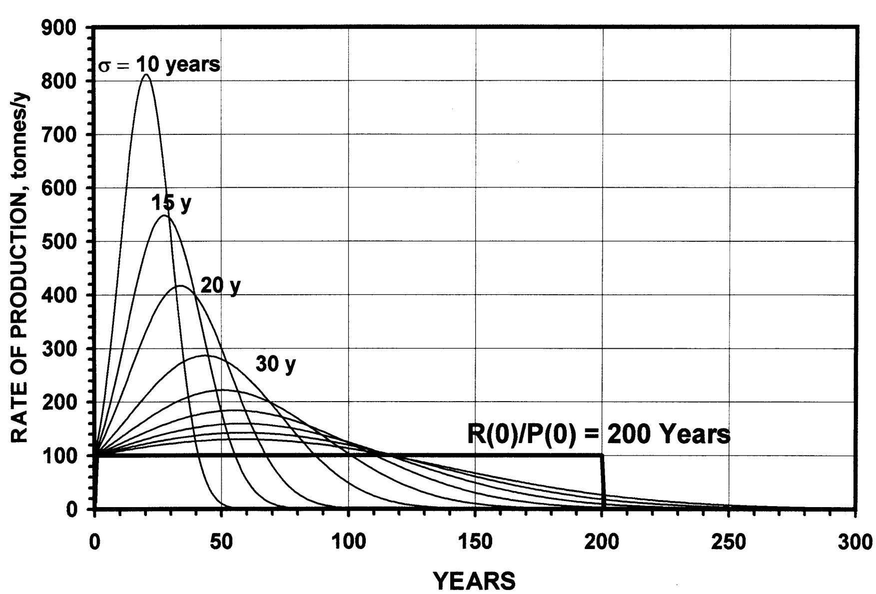

Figure 4. Generic graph of curves of $P(t)$ vs. t for case of $R(0)/P(0) = 200$ years. Horizontal curve shows constant $P(t)$ of 100 tonnes/y for 200 years at which point resource is consumed completely and $P(t)$ falls abruptly to zero. Other curves are "business as usual" Gaussian scenarios for values of standard deviation σ of curves of (from top to bottom) 10 y, 15 y, 20 y, 30 y, 40 y, 50 y, 60 y, 70 y, and 80 y. Curves all have same area of $R(0) = 20{,}000$ tonnes. Numerical data from these curves are given in Tables 7 and 8.

Table 3. For the Initial Reserve to Production Ratio, $R(0)/P(0)$ of 50 Years, and $P(0) = 100$ Tonnes/y, as Shown in Figure 2, This Table Gives the Expected Values of Four Features of the Gaussian Curves that Represent the "Business as Usual" Scenarios of $P(t)$ vs. t for Different Values of σ

σ Sigma in Years	Initial Slope at $t = 0$% Per Year (%)	Year of Gaussian Peak	Height of the Gaussain Peak Tonnes/Year	Year of Return to Original $P(0)$ 100 tonnes/y
10	14.3	13	220	25
15	6.8	14	158	29
20	3.7	14	129	29
25	2.1	13	114	25
30	1.1	10	105	19
35	0.42	5	101	10

For example, for $\sigma = 20$ years, the initial slope $(1/P)dP/dt = +3.7\%$ per year. The Gaussian peak will be reached in 14 years at which time $P(14) = 129$ tonnes/y. At $t = 29$ y, $P(29) = 100$ tonnes/y and by symmetry $(1/P)dP/dt = -3.7\%$ per year.

for Australian LNG would be reached in 17 years at which time P would be 4.16 $P(0)$ and the R/P ratio would then be 13 years. In 34 years P would fall to its initial value of 100 tonnes/y at which time the R/P ratio would be about 5.2 years. The curve for $\sigma = 10$ years in Figure 3 is a graphic portrayal of the approximate expected "business as usual" scenario for the future production of Australia's LNG, based on the numbers given by the Minister.

CALCULATION OF *R/P* RATIOS AS A FUNCTION OF TIME

The R/P ratio can be calculated as a function of time for every point on each of the Gaussian curves

Table 4. For the Reserve to Production Ratio $R(0)/P(0) = 50$ Years, as shown in Figure 2, This Table Gives, for Several Values of σ, the Number of Years Needed to Consume Half of $R(0)$, the R/P Ratio at the Peak of the Gaussian Curve, the R/P Ratio after the Peak has been Passed and $P(t)$ has Declined Back to its Initial Value of 100 Tonnes/Year

σ Sigma in Years	Years to Consume Half of $R(0)$	R/P in Years at Peak	R/P at in Years When $P(t) = P(0)$
10	14	12.6	6.32
15	18	19.7	10.6
20	21	26.1	16.0
25	23	31.9	22.9
30	25	38.1	30.7
35	26	44.9	40.3

Table 5. For the Initial Reserve to Production Ratio, $R(0)/P(0)$ of 100 Years, and $P(0) = 100$ Tonnes/y, as Shown in Figure 3, This Table Gives the Expected Values of Four Features of the Gaussian Curves that Represent the "Business as Usual" Scenarios of $P(t)$ vs. t for Different Values of σ

σ Sigma in Years	Initial Slope at $t = 0$% Per Year (%)	Year of Gaussian Peak	Height of the Gaussain Peak Tonnes/Year	Year of Return to Original Rate of Production
10	20.1	17	416	34
15	10.7	22	286	44
20	6.72	25	221	50
30	3.32	29	159	58
40	1.84	29	130	58
50	1.04	26	114	51
60	0.55	20	106	39
70	0.23	11	101	23

For example: for $\sigma = 20$ years, the initial slope $(1/P)dP/dt = +6.72\%$ per year. The Gaussian peak will be reached in 25 years at which time $P(25) = 221$ tonnes/y. At $t = 50$ y, $P(50) = 100$ tonnes/y and by symmetry $(1/P)dP/dt = -6.72\%$ per year.

of $P(t)$ vs. t and, as expected, the R/P ratio, based on the current estimate of $R(0)$, decreases continuously with increasing time as is shown in Figure 5 for the case of $R(0)/P(0) = 200$ years. These curves are similar to those shown by Bartlett (2000, p. 14) for the curves for the R/P vs. time for Gaussian curves for world oil. The reasonableness of the values of R/P is indicated by the fact that the current R/P for world oil is often stated to be approximately 40 years which is in good agreement with the value from Bartlett's R/P curve for $R(i) = 2 \times E(12)$ barrels.

Table 6. For the Reserve to Production Ratio $R(0)/P(0) = 100$ Years, as Shown in Figure 3, This Table Gives, for Several Values of σ, the Number of Years Needed to Consume Half of $R(0)$, the R/P Ratio at the Peak of the Gaussian Curve, and the R/P Ratio After the Peak Has Been Passed and $P(t)$ Has Declined Back to its Initial Value of 100 Tonnes/Year

σ Sigma in Years	Years to Consume Half of $R(0)$	R/P in Years at Peak	R/P in Years When $P(t) = P(0)$
10	18	13	5.2
15	23	19	8.5
20	28	26	12
30	35	38	21
40	41	51	31
50	45	63	44
60	49	75	59
70	51	88	77

Table 7. For the Initial Reserve to Production Ratio, $R(0)/P(0)$ of 200 Years, and $P(0) = 100$ Tonnes/y, as Shown in Figure 4, This Table Gives the Expected Values of Four Features of the Gaussian Curves that Represent the "Business as Usual" Scenarios of $P(t)$ vs. t for Different Values of σ

σ Sigma in Years	Initial Slope at $t = 0$ % Per Year (%)	Year of Gaussian Peak	Height of the Gaussian Peak in Tonnes/Year	Year of Return to Original $P(0)$
10	20.5	20	812	41
15	12.3	28	548	55
20	8.4	34	417	68
30	4.8	44	286	87
40	3.2	51	222	101
50	2.2	55	184	110
60	1.6	58	159	116
70	1.2	59	142	117

For example: for $\sigma = 20$ years, the initial slope $(1/P)dP/dt = +8.4\%$ per year. The Gaussian peak will be reached in 34 years at which time $P(34) = 417$ tonnes/y. At $t = 68$ y, $P(68) = 100$ tonnes/y and by symmetry $(1/P)dP/dt = -8.4\%$ per year.

ENVELOPE OF CURVES OF RATES OF PRODUCTION

On each of the Figures 2–4, a smooth envelope curve tangent to the right sides of the family of peaks can be drawn. Such an envelope curve for $R(0)/P(0) = 200$ years is shown in Figure 6 as the upper curve AA. Similar curves BB and CC are shown for $R(0)/P(0)$ values respectively of 100 years and 50 years. All of the Gaussian (business as usual) trajectories of $P(t)$ vs. t for a given $R(0)/P(0)$ are confined to the region below the appropriate envelope curve. The envelope curve divides the plane of the graph into two areas, an "allowed" area below and the "forbidden" area above the envelope curve. The separation of the graph into two areas is not absolute. One could gerrymander a hypothetical curve of $P(t)$ vs. time, such as those shown in Figure 1, that rises rapidly and suddenly collapses to zero. Such a curve could rise briefly into the forbidden area, but the "business as usual curves" won't have any excursions above the envelope curve. For example, one can look at the envelope curve in Figure 6 and say that for $R(0)/P(0) = 200$ years (curve AA), the "business as usual scenarios" will not permit the resource to be produced at a rate of 200 tonnes/y at $t = 100$ years.

Table 8. For the Reserve to Production Ratio $R(0)/P(0) = 200$ Years, as Shown in Figure 4, This Table Gives, for Several Values of σ, the Number of Years Needed to Consume Half of $R(0)$, the R/P Ratio at the Peak of the Gaussian Curve, and the R/P Ratio After the Peak Has Been Passed and $P(t)$ has Declined Back to its Initial Value of 100 Tonnes/Year

σ Sigma in Years	Years to Consume Half of $R(0)$	R/P in Years at Peak	R/P in Years When $P(t) = P(0)$
10	21	13.5	4.65
15	29	19	7.28
20	35	25.4	9.94
30	47	38.1	16.3
40	56	50.5	23.5
50	64	63.4	31.7
60	71	71.8	40.5
70	77	88	50.6

SUSTAINABILITY

The definition of sustainable development (sustainability) is taken from Bruntdland (1987):

> Sustainable development is development that meets the needs of the present without compromising the ability of future generations to meet their own needs.

Sustainability has to imply "for periods of time long compared to a human lifetime." It is clear that nonrenewable resources that we consume today will not be available for the use of future generations. *This has the clear consequence that consumption of nonrenewable resources is not sustainable, a fact that is rarely acknowledged by sustainability "experts."* Figure 1 shows that steady growth in the rate of production of a resource is not sustainable. It follows that when one is dealing with real material things the term "*sustainable growth*" is an *oxymoron*. It is clear from Figures 2–4 that the scenario of producing the resource "at the present rates of production" is not sustainable. It is also clear from Figures 2–4 that the "business as usual" scenarios of $P(t)$vs. t all rapidly approach zero asymptotically, so these scenarios leave little production for future generations.

Is there any way we can consume resources today without seriously compromising the ability of future generations to meet their own needs? Can we eat our cake and have it too?

The mathematics provides a qualified "yes." Suppose $P(t)$ declines steadily according to the relation:

$$P(t) = P(0)\ \exp(-kt). \tag{5}$$

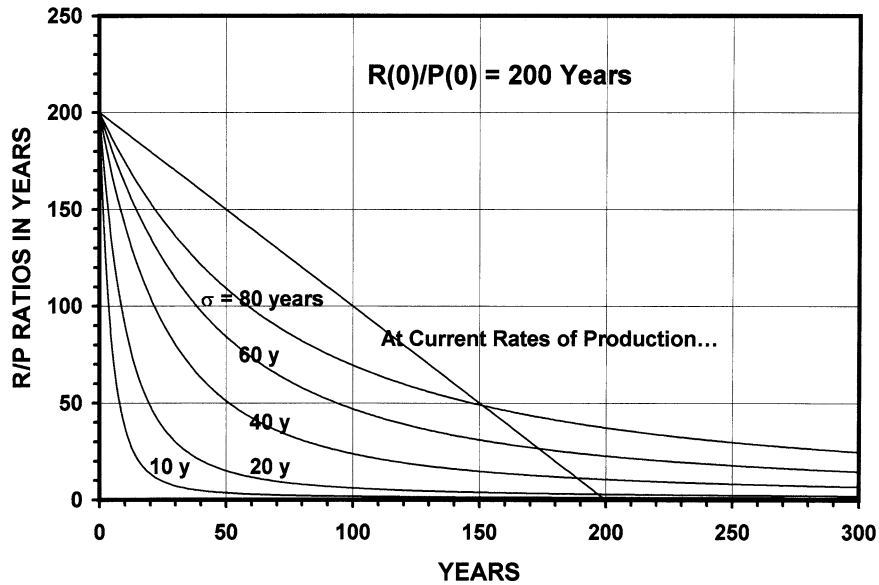

Figure 5. Graph showing evolution with time of Reserve to Production ratio, R/P, for scenario "at current rates of production" (declining straightline) and "business as usual" hypothetical Gaussian curves, for values of standard deviation σ of (from top to bottom) 80 y, 60 y, 40 y, 20 y, and 10y, all for initial value of $R(0)/P(0) = 200$ years. The smaller the value of sigma, the higher and narrower is the Gaussian peak and the more rapid is the decline of the R/P Ratio.

The total quantity of the resource remaining at $t = 0$ is $R(0)$ which must be equal to all the resource that will be extracted in all future time. All future consumption is given by the integral from zero to infinity $P(0) \int \exp(-kt)\, dt$. This integral has the value $P(0)/k$ which must equal $R(0)$. From this we can define the "sustainability coefficient" $k(s)$ to have the value:

$$k(s) = P(0)/R(0) \qquad (6)$$

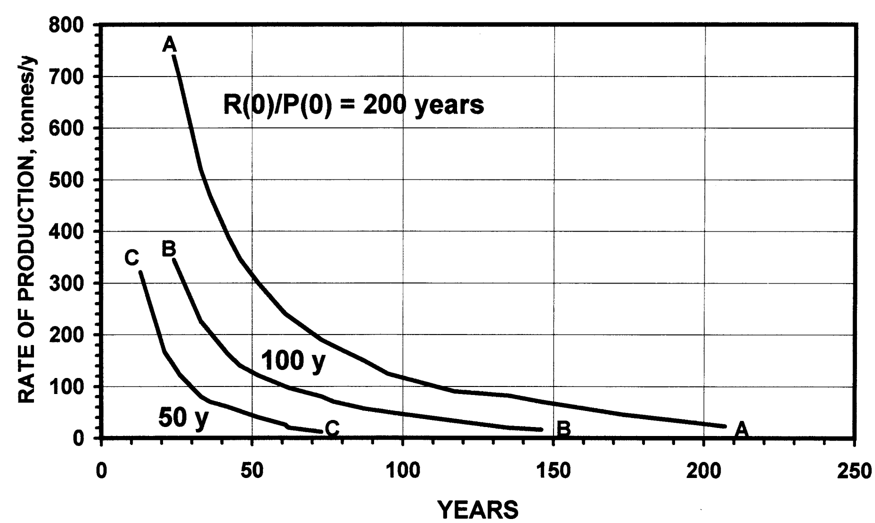

Figure 6. Curves showing envelopes of "business as usual" Gaussian curves for three values of ratio $R(0)/P(0)$, 50, 100, and 200 y. For given value of $R(0)/P(0)$ area above corresponding curve is "Forbidden" and area below curve is "Allowed." Looking at curves one can say that if present value of $R(0)/P(0)$ is 100 years, then "business as usual" scenarios can not produce a rate of production of 200 tonnes/y 50 years from now but could produce 100 tonnes/y.

The sustainability coefficient $k(s)$ is the inverse of the $R(0)/P(0)$ ratio. When $P(t)$ declines according to Equation (5) with k having the value given by Equation (6), $P(t)$ can continue forever on this constantly declining curve! For example, if $R(0)/P(0) = 200$ years, then the sustainability coefficient $k(s) = P(0)/R(0) = 1/200 = 0.005$ per year (half a percent per year). If $R(0)/P(0) = 100$ years, then a curve of $P(t)$ that declines 1% per year is sufficient to allow the resource to last forever! If $P(t)$ declines more rapidly than given by this sustainability coefficient, then $P(t)$ can continue forever without consuming all of the resource.

> If a resource will last $R(0)/P(0)$ years at present rates of production, and if the rate of production of the resource follows the curve that starts at $P(0)$ and has a constant fractional decrease per unit time whose magnitude is greater than or equal to $P(0)/R(0)$ per year, then the production can be truly said to be sustainable.

This is probably as close as it is possible to come to the Brundtland definition of sustainability for the use of a nonrenewable resource. This path provides a measure of real, but declining, intergenerational equity in the use of finite reserves of nonrenewable resources for all future generations.

This concept has been given the name "Sustained Availability" (Bartlett, 1986) and it has other interesting characteristics:

(1) At the time $R(0)/P(0)$, the rate of production has declined to $1/e = 0.368\ldots$ of its initial value $P(0)$. For example, for $R(0)/P(0) = 200$ years, P would decline to 36.8% of its initial value in 200 years.
(2) For any current value of $R(0)/P(0)$ one can calculate the sustainability coefficient $k(s)$ needed to define the sustainable curve from that point forward. If changes are made in $R(0)$ through new discoveries or through corrections of earlier estimates of $R(0)$, one can calculate the new value of $k(s)$ and thereafter proceed down the new declining exponential curve.
(3) At all points on a declining exponential curve, the R/P ratio is unchanged from its initial value of $R(0)/P(0)$.

The mathematics of this curve and the options it presents for a producing country that consumes part of its production and exports the balance of its production, have been explored in detail (Bartlett, 1986).

FORBIDDEN, TERMINAL, AND SUSTAINABLE PRODUCTION PATHS

We now can add the declining exponential curve to the graph of Figure 6 as is shown in Figure 7 for the case of $R(0)/P(0) = 100$ years. The two lines divide the plot into three areas. The area in the upper right above the curve AA is "*F*" for "forbidden," because the "business as usual" scenarios of $P(t)$ vs. t cannot appear in this area. Between the envelope curve AA and the declining exponential curve BB is an area labeled "*T*" for "terminal," in which all of the "business as usual" Gaussian scenarios of $P(t)$ vs. t are possible but they approach zero more rapidly than the declining exponential curves, BB, and hence are effectively terminal. Below the curve BB is the area "*S*" for "sustainable." Any scenario of $P(t)$ vs. t that starts at $P(0)$ and stays below curve BB is sustainable.

Figure 7 shows that the options for sustainability are limited. This fact is not appreciated by governmental, industrial, educational, scientific, environmental, and policy leaders, many of whom seem to be dedicated to unending economic growth *and* sustainability, hoping perhaps that marvels of technology will arrive in time to remove the limits dictated by nature that are outlined here. Indeed, a friend told me of attending an international conference on sustainability where he heard people saying words to the effect that "We're tired of hearing of limits. We're going to grow the limits!"

AUSTRALIA'S ENERGY POLICIES AND THE LIMITATIONS IMPOSED BY NATURE

A number of troubling things come to mind when one reads the article written by the Australia's Minister of Industry, Tourism and Resources from which the opening statement was taken.

(1) The Minister's optimistic assessment of Australia's energy situation is based on the Government's major long-range energy plan. Thus, the misrepresentations set forth by the Minister and cited here, are probably built into Australia's National Energy Plan.

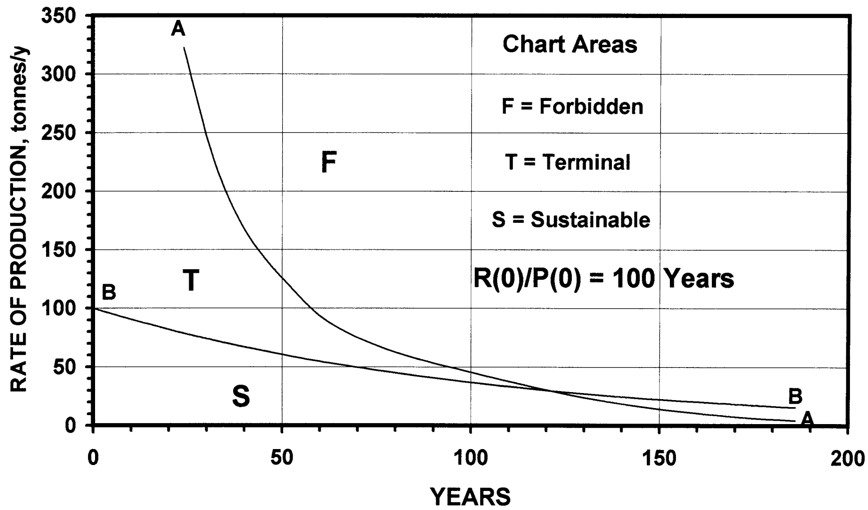

Figure 7. (P, t) plane is divided into three areas by two lines, AA and BB as shown here for $R(0)/P(0) = 100$ years. Line BB is curve of sustainability in which $P(t)$ decreases at constant fractional rate of 1% per year. If $P(t)$ starts at 100 tonnes/y and stays on or below this curve, resource can be produced theoretically at declining rates forever. Line AA is envelope of "business as usual" Gaussian curves. Region above line AA is "F" for "forbidden." Region between lines is "T" for "terminal" region. All "business as usual" Gaussian curves of $P(t)$ vs. t whose initial slope is positive will start in "T" region and will ultimately fall below line BB and will approach zero more rapidly than "sustainable" declining exponential curve BB.

(2) The Minister made no mention of the growth of Australia's population. This suggests that the Government's long-range energy plan does not concern itself with the size of Australia's population or with its rate of growth. If this is the situation, the omission dooms the Government's entire plan. Any responsible governmental policy for "Australia's long-term development, security and environmental needs as they relate to energy" has to take full account of Australia's continuing population growth and of the predictable effects this growth will have in forcing the future growth of Australia's domestic energy consumption. In the face of the acknowledged finite nature of fossil fuels and the predictable growing domestic demand for energy, the Minister recounts with pride that Australia is exporting its fossil fuels as rapidly as possible, presumably in accord with the Government's plan.

> "I am expecting and planning for Australia to further expand its energy export base over the coming decade and beyond."

It is probable that before mid-century the people of Australia will be needing the fossil fuels that their government is so eagerly exporting.

(3) In something like a century Australia's population has increased by a factor of 5 from about 4 million to 20 million. Equation (3) shows that this enormous increase represents an average growth rate of only 1.6% per year! Another century of 1.6% per year growth would cause Australia's population to increase by another factor of 5 to 100 million!

(4) Malkin (2005) describes the *City of Cities* plan for the City of Sydney, reporting that in the next 25 years Sydney will construct 447,000 new dwellings and create almost 300,000 new jobs. Even though this projected growth violates the Laws of Sustainability (Bartlett, 1994), Sydney's Planning Minister makes the amazing suggestion that this will be done carefully "to ensure the city grows in a sustainable way." Other Australian cities may have similarly plans for enormous population expansions.

(5) Contemporary reports tell of the shortage of potable water in Australian cities which

has led to plans to build large plants for the desalination of sea water. These plants are just one of the many predictable costly infrastructure developments that will be needed to accommodate the projected large increases in population, all of which will require substantial annual consumption of energy. Here is a Greek tragedy unfolding. The Government's policy is one of getting rid of (exporting) the nation's fossil fuels as rapidly as possible and at the same time the Government seems to be promoting, or accepting, long-term rapid growth of Australia's population. Any conservative plan for Australia's future would recognize that this projected population growth will require the fossil fuels that now are being exported. Yet the Minister writes that the "government's white paper on energy" from which these policies flow, "is a substantive and forward-thinking document that allows us to maintain some of the world's lowest energy costs while also reducing our greenhouse signature." This policy is not "forward-thinking" for the reason that it ignores the simple confluence of the arithmetic of population growth and the arithmetic of resource reserves and consumption. The Government's policy is not even "backward-thinking" because it ignores the recent history which makes clear the growing magnitude of the quantitative problems that flow from continued population growth. And, as is shown here, the policies described by the Minister do not decrease Australia's "greenhouse signature," they *increase* it.

(6) The Minister opens his article by accepting the premise that. "The demand for world energy is growing and will continue to grow for the foreseeable future." Demand may grow, but there are many indications that the world's fossil fuel resources will, in the near future, fall short of meeting this growing demand. This raises a fundamental question: Is it the obligation of countries that presently have reserves of fossil fuels, to export these fuels to the importing countries where the demand is already larger than the importing countries can meet from their own domestic supplies?

(7) The title of the Minister's article, "A Growing Role for Australia in Meeting the World's Energy Needs" suggests that Australia is accepting an obligation to export its fossil fuels as rapidly as possible. This violates the Brundtland statement of sustainability because it is clear that this cannot be done without "compromising the ability of future generations of Australians to meet their own needs." The Government's policy set forth by the Minister is not sustainable; quite the opposite, it's "anti-sustainable." The policy is the equivalent of shooting yourself in the foot.

(8) The life-expectancy of Australian coal of 110 years "at current rates of production" is alarmingly short compared to the life-expectancy of a major nation. *A thoughtful response to this datum by the Government would be alarm, coupled with a call for measures to reduce the rate of production of Australian coal well below the present level so that reserves of the resource will be available for Australians beyond 110 years. A responsible government policy would be to call for a halt to the export of Australian energy resources so that the coming generations of Australians will be able to have the benefits of the use of these precious resources.*

(9) The Minister is proud to boast of how long the resources would last "at current rates of production," but it makes clear that the Government's policies are not to maintain the "current rates of production" but rather to have rapid growth of the rates of production. This is misleading in the extreme.

(10) There seems to be no recognition of the obvious, but inconvenient, truth that growth of the rates of production reduces the life-expectancy of finite reserves of nonrenewable resources far below the figure given by the frequently cited $R(0)/P(0)$ ratio.

(11) The Government professes a devotion to sustainability but its policies violate the First and Second Laws of Sustainability:

First Law: "Population growth and/or growth in the rates of consumption of resources cannot be sustained."

Second Law: "In a society with a growing population and/or growing rates of consumption of resources, the larger the population and/or the larger the rates of consumption of resources, the more difficult it will be to transform the society to the condition of sustainability." (Bartlett, 1994)

WHO SHOULD BE RESPONSIBLE FOR GREENHOUSE GASES?

On two counts, it is simply not true that the Government's policies are "reducing our greenhouse signature." *By accepting domestic population growth, Australia will almost certainly be increasing its domestic production of greenhouse gases. By exporting coal and LNG to be burned in other countries, Australia is making a significant further contribution to the increase of global atmospheric greenhouse gases*.

This raises a fundamental policy question. If country A annually exports a quantity of fossil fuels which are burned in country B to produce C tonnes of greenhouse gases, which country should be held responsible for the introduction of these C tonnes of greenhouse gases into the Earth's atmosphere? A good case can be made for holding each country responsible for the full amount of C. Affirmative decisions are required by both countries A and B in order to introduce the greenhouse gases into the global atmosphere. Country A can decide not to export the fossil fuels, and country B can decide not to import them. Consequently, both countries A and B should be held responsible for the C tonnes/y of greenhouse gases.

SHORT-TERM POLITICIANS AND LONG-TERM PROBLEMS

Politicians live for the short term, and one can see on Figures 2–4 that for a decade or two it will be possible to have Australia's production of fossil fuels follow the growth curves that make business people and politicians happy. In business and governmental circles, growth is the universally accepted measure of success, and for this reason politicians can follow the "business as usual" scenarios in these early growth periods and can later claim that they have had great success in boosting the growth of their country's economy. But governmental leaders seem not to recognize that in the near future, growth will lead to the peak of the rate of production. *Government actions can delay the peak but there are no governmental actions that can eliminate or avoid the peak.* After the peak, there will be hardships.

Although the example cited here is from Australia, similar examples abound in all of the industrialized nations of the world. Breathtaking innumeracy is an integral part of governmental planning and actions throughout the "educated" world. Journalists rarely point out this innumeracy because, for the most part, they too are innumerate.

INTERNATIONAL TRADE AGREEMENTS

International trade agreements are supported ardently by the governments of the developed nations which largely have consumed their domestic fuels and which are consequently dependent on imports of fossil fuels. One can reasonably presume that a major motivation for these currently popular international trade agreements is to let the developed nations get their hands on other nations' resources before the people of the other nations develop to the point of needing their own resources. This contemporary colonialism has led to widespread environmental degradation and it has contributed to the growing economic disparity between the rich and the poor (Eherenfeld, 2005). These trade agreements need to be re-examined. *The international trade agreements are an impediment to sustainability if they prohibit a nation from following the conservative steps of taking its exportable natural resources off of the international market to save the resources for future domestic use.*

POLICY RECOMMENDATIONS

It is essential that the people of Australia and of other nations similarly situated;

(1) Stop their population growth and encourage their populations to decline to sustainable levels;
(2) Stop the export of their nonrenewable natural resources, especially fossil fuels;
(3) Put the production of domestic resources on the declining exponential "sustainability curves."
(4) Improve the efficiency with which all resources are used;
(5) Develop their domestic renewable energy resources as rapidly as possible.

All of these policy recommendations should be central to any governmental plan that purports to "Secure [the] Energy Future" of its people.

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